Sample Final Exam

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Problem I :

Consider the following ODE :

$$\frac{dy}{dx} = a(x)y + b(x)y^2 + c(x)$$

a) Assume $a = 1, b = 2, c = -3$ find a solution to the ODE

b) Assume $a = 0$ and that $c(x) = x^n$ solve the ODE for $n = -2$

[Hint : use the substitution $y = \frac{1}{z}$]

c) If a particular solution y_1 is known , the general solution is given by $y = y_1 + u$ show that

$$\frac{du}{dx} = b(x)u^2 + [2b(x)y_1 + a(x)]u$$

Problem II :

Solve the following ODEs :

a) $x't(x' + 2) = x$

b) $y(2x^4 + y)\frac{dy}{dx} = (1 - 4xy^2)x^2$

c) $y'' + y = 8\cos(x)\cos(2x)$ by :

i) knowing that cos(a)cos(b) = $\frac{\cos(a-b)+\cos(a+b)}{2}$ and using variation of parameters

ii) using the Wronskian find the general solution

d) $y'' - 2y' + 2y = x + 1$ y(0) = a ; y'(0) = 0

Problem III :

The following ODE is not exact . **<u>DERIVE</u>** an integrating factor to make it exact :

$$(2x+1)(\cos(\mathrm{y}))\mathrm{dx} + (\mathrm{x}^2 + x)(\sin(y)) + \tan(y)\, dy = 0$$

Problem IV :

Use $x = e^t$ to convert the Cauchy-Euler equation into a differential equation with constant coefficients then solve the ODE :

$$x^2 y'' - xy' + y = \ln(x)$$

Problem V :

i) Using the method of Frobenius solve the ODE :

$$3xy'' + y' - y = 0 \quad \text{about } x = 0$$

ii) Using the taylor series of cos(x) solve the ODE :

$$y'' + (\cos(x))y = 0$$

Problem VI :

Using Laplace Transforms solve the following ODEs :

a) $y'' + 3y' + 2y = g(t)$; $y(0) = 0 \; and \; y'(0) = 1$

$$g(t) = \begin{cases} 1 & if \; 0 \le t < 1 \\ 0 & if \; t \ge 1 \end{cases}$$

b) $y'(t) = 1 - \int_0^t y(t-v)e^{-2v}dv$

c) $y'' + 2y' - 3y = 6t^2e^{-3t}$ [express in convolutions]

Problem VII :

Using eigenvalues solve the ODEs :

a) $X' = \begin{pmatrix} 3 & 6 \\ -1 & -2 \end{pmatrix} X$

b) $X' = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix} X$

c) $X' = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix} X$

Problem VIII :

Think of a way to solve this first order ODE :

$$\frac{dy}{dx} = \frac{1}{x(x-y)}$$